GASDYNAMIC MODEL OF THE FLOW OF AN INVERSE MIXTURE OF GASES IN A RESONATOR

V. M. Kuznetsov

UDC 533.6.011

To calculate the flow of an inverse mixture of multiatomic gases in a resonator taking into account all the gas dynamic and optical phenomena is a complex and unsolved problem. Several simple one-dimensional models exist [1, 2], which are based, in particular, on the constant-gain approximation. On the basis of these models the condition for steady-state lasing is $2k^*d = -\ln r_1r_2$ [3], where k^* is the saturated gain, d is the distance between the resonator mirrors, and r_1 and r_2 are the reflection coefficients. Since the gain of the active medium at the input of the resonator (x = 0) $k_\infty \neq k^*$, a discontinuity occurs in the values of the required variables. This discontinuity can be eliminated by introducing a thin transition region in which either the value of one of the reflection coefficients r = r(x) is assumed to be variable [1], or a certain relation is introduced for the radiation intensity I = I(x) [2], which ensures a transition from the gain k_∞ to k^*. Further along the flow the set of equations of a relaxing mixture of gases is solved with the condition $k(x) = k^*$. Obviously there is a certain amount of arbitrariness in choosing any "matching" relations. We will show that an analytical solution of the problem can be obtained within the framework of the model considered. Note that analytical models for gasdynamic CO_2 lasers have been constructed previously [4-6]. These models describe the physics of the processes occurring in the resonator fairly well, and enable one to calculate the output power. However, the transition region at the input to the resonator is not always considered, and the effect of radiation on the change in the gasdynamic parameters of the flow is ignored. The model assumed in this paper also enables one to take these features into account. In addition, in the approach adopted here the problem of calculating the structure of the flow in the resonator is considered as a problem in perturbation theory, which has much in common with calculations of the structure of a shock wave in a relaxing mixture of $CO_2 + N_2 + H_2O$ (He) [7]. This follows, first, from the fact that for a plane Fabry–Perot interferometer the problem remains one-dimensional since across the field of flow the radiation intensity in a high-Q resonator is constant (for $r_1, r_2 > 0.5$) [8]. Second, to solve the problem one can use the conservation equation in integral form [9]

$$\rho u = \rho_\infty u_\infty = C_1, \quad p + \rho u^2 = p_\infty + \rho_\infty u_\infty^2 = C_2, \tag{1}$$
$$\frac{u^2}{2} + c_pT + \sum_{i=1}^{N}\beta_iE_i + I/\rho_\infty u_\infty = \frac{u_\infty^2}{2} + c_pT_\infty + \sum_{i=1}^{N}\beta_iE_{i\infty} = C_3,$$
$$p = \rho\frac{R}{\mu}T,$$

where ρ, u, p, and T are the density, velocity, pressure, and temperature of the mixture; μ, β_i, molecular weight and the molar fraction of the i-th component of the mixture; E_i, internal energy of the individual modes of oscillation; and I = I(x), a function representing the intensity distribution of the radiation. Up to the section with index zero (the input to the resonator) I = 0 and the constants C_1, C_2, and C_3 are known from the solution of the problem of the expansion of the mixture of gases (e.g., for the discharge from a nozzle). For the closure of system (1) we need equations for the unknown functions E_i and I. The relaxation equations for E_i are chosen in the Landau–Teller form for each mode of oscillation [7]. Intermode exchanges by quanta in this model are taken into account approximately in terms of the effective oscillatory relaxation time, as in the Anderson model [8].

In laser mixtures based on CO_2 the ratio of the total oscillatory energy to the total damping enthalpy H_∞ is the small parameter $\varepsilon = \sum_i \beta_iE_i/H_\infty \ll 1$ [7]. Since the radiation energy

---

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 29-33, January-February, 1980. Original article submitted October 17, 1978.

0021-8944/80/2101-0025$07.50

is part of the oscillatory energy luminescing from certain levels, the ratio $\varepsilon = \left(\sum_i \beta_i E_i + I/\rho_\infty u_\infty\right)/H_\infty$ is also small. In view of this we are dealing with a typical problem in perturbation theory, when, to a first approximation, we can assume that ε has no effect on the density distribution, the pressure p etc., found from the conservation equations (1) for $\varepsilon = 0$ [10, 7]. The values of p_∞, ρ_∞, u_∞, T_∞ obtained in this way are used to solve the relaxation equations irrespective of the remaining equations of hydrodynamics for $\varepsilon = 0$. If we introduce the mean number of quanta for each mode $e_i = [\exp(\Theta_i/T_i) - 1]^{-1}$, where $\Theta_i = h\nu_i/k$ is the characteristic temperature, the equations for modes i = 1,3 in the resonator take the form

$$\frac{de_1}{dx} = \frac{e_1(T_0) - e_1(T_1)}{u_\infty \tau_1} + \frac{kI}{u_\infty n_\infty h\nu\beta_{CO_2}},$$
$$\frac{de_3}{dx} = \frac{e_3(T_0) - e_3(T_3)}{u_\infty \tau_3} - \frac{kI}{u_\infty n_\infty h\nu\beta_{CO_2}}. \tag{2}$$

Here and henceforth the notation used is the one generally employed [8]. Since at the inputs of the resonator the frequency of the stimulated radiational transitions exceeds the frequency of the collisions which deactivate the upper laser level, the latter can be neglected in the region $0 \leqslant x \leqslant \delta$.*

Henceforth, we will use the expression for the gain at the center of the line with the simplifications made in [8]. It can then be represented in the following form: $k = C(e_3 - e_1)$, where $C \approx \text{const}$, and $e_i \ll 1$. The quantities $e_i(x = \delta) = e_i^*$ can be found from the condition $C(e_3^* - e_1^*) = k^*$ and by preserving the number of oscillatory quanta $e_1^* + e_3^* = (e_1 + e_3)_{x=0} = e_1(x) + e_3(x) = M = \text{const}$, where M is known from the solution up to the resonator input. Using these conditions and Eq. (2) without the collisional terms, we can determine the contribution to the radiation power made by the transition zone

$$P_1 = \frac{tb}{1+r} \frac{u_\infty n_\infty h\nu\beta_{CO_2}}{C} \ln \sqrt{\frac{k_\infty}{k^*}}, \tag{3}$$

where t and b are the transmittance and width of the mirrors, $r_1 = 1$, $r_2 = r = 1 - \alpha - t$, and α is the loss coefficient, i.e., for simplicity we assume that the losses and the radiation output are concentrated on one mirror. In the main region we have the condition $k^* = \text{const}$, i.e., $dk^*/dx = 0$. Then it follows from the expression for the gain that $de_3/dx \approx de_1/dx$, since to a first approximation, i.e., for $\varepsilon = 0$, $p = p_\infty$, $u = u_\infty$, $T = T_\infty = T_0$ etc., everywhere inside the resonator and $C \approx \text{const}$. In the equilibrium state $e_1 = e_{10}(T_0)$, but since we have imposed the additional condition $k^* = C(e_{30} - e_{10}) = \text{const}$ on the solution, the equilibrium value of the oscillatory energy of the third mode will be $k^*/C + e_{10}$.

The main part of the useful power P is concentrated in the nitrogen molecules, with the exception of a part P_1(3), related to the transition zone. In this connection, to estimate P we will consider the relaxation equations only for the modes i = 3 and i = 4, while the population of the lower levels in the modes i = 1,2 will be neglected [5, 6] due to their rapid relaxation. Then in the steady-state lasing mode $e_3 = e_3^* = \text{const}$ everywhere inside the resonator for p = const and T = const. On the other hand, computer calculations show [2, 8], that for $k^* = \text{const}$ the condition $de_3/dx \approx de_1/dx$ is in fact realized. The linear model [2], strictly speaking, is extremely rough for describing relaxation processes inside the resonator. However, we would expect to be able to obtain a correct estimate for the radiation power P which is an integral characteristic. For this purpose we will normalize the terms with radiation in system 2 with respect to the total concentration $n_{CO_2} + n_{N_2} = n_\infty \beta_{CO_2 + N_2}$. We then obtain a model mixture in which the nitrogen is formally replaced by CO_2, but the relaxation of which is characterized by the parameters of the gases of the initially chosen composition.

*In the simplest model [4] the collisional terms are neglected in the whole region of the flow assuming that the length of the resonator $L \ll u_\infty \tau_1$.

Solving the system of relaxation equations (2) taking these factors into account, we obtain

$$e_3(x) = \left(\frac{k^*}{C} + e_{10}\right) + \frac{2k^* I(x)}{\beta_{CO_2+N_2} u_\infty n_\infty h\nu}\left(\frac{1}{u_\infty \tau_3} - \frac{1}{u_\infty \tau_1}\right)^{-1}, \tag{4}$$

where

$$I(x) = \frac{\beta_{CO_2+N_2} u_\infty n_{CO_2} h\nu\,(e_1^* - e_{10})}{2k^*}\left(\frac{1}{u_\infty \tau_1} - \frac{1}{u_\infty \tau_3}\right) e^{-Dx}; \tag{5}$$

$$D = \frac{1}{2}\left(\frac{1}{u_\infty \tau_1} + \frac{1}{u_\infty \tau_3}\right).$$

The expression for the output power can be represented in the form

$$P_2 = \frac{tb}{1+r}\,\frac{u_\infty n_\infty h\nu \beta_{CO_2+N_2}(e_1^* - e_{10})}{2k^* D}\left(1 - e^{-Dx}\right)\left(\frac{1}{u_\infty \tau_1} + \frac{1}{u_\infty \tau_3}\right). \tag{6}$$

Since $u_\infty \tau_3 \gg u_\infty \tau_1$, the term $1/u_\infty \tau_3$ can be neglected. This can in fact be done for any resonator length L since because of the condition k^* = const assumed above the modes i = 3,4 are either damped [6, 8], or approach the above-mentioned state $e_{30} = k^*/C + e_{10}$ which is not the usual equilibrium state related to the collisional deactivation in the $\sim u_\infty \tau_3$† scales. Then, for x = 0 and $\beta = \beta_{CO_2}$, using the energy equation (1) we obtain the following estimate of δ:

$$\delta \sim \frac{(e_1^* - e_{1\infty})\, d}{(e_1^* - e_{10})\, 2k^* \tau_1 C\alpha}.$$

If the resonator is not very short ($L \gg u_\infty \tau_1$) we can neglect the last term in expression (6), and we then obtain with respect to the total output

$$\mathscr{P} = \frac{P - P_1}{\rho_\infty u_\infty S} = \frac{t}{1+r}\,\frac{h\nu N_A (e_1^* - e_{10})\,\beta_{CO_2+N_2}}{k^* d \mu}, \tag{7}$$

where N_A is Avogadro's number.

Hence, it turns out that to estimate the specific power the following information on the flow is sufficient: For a specified resonator geometry and the percentage composition of the mixture it is necessary to know the temperature T in the resonator in order to calculate e_{10} and the initial data for calculating e_1^*. It is not necessary to know the constants of the elementary processes and we only need to satisfy the inequality $u_\infty \tau_3 \gg u_\infty \tau_1$. For very short resonators ($L \leqslant u_\infty \tau_1$) it is necessary to calculate the quantity $u_\infty \tau_1$.

In order to estimate the effect of the radiation on the change in the gasdynamic parameters in the resonator it is necessary to consider the following approximation with regard to ε, i.e., to obtain the quantities $p' = (p - p_\infty)/p_\infty$, $\rho' = (\rho - \rho_\infty)/\rho_\infty$ etc. The latter can be found by linearizing system (1) with respect to ε, and have the form

$$p' = \frac{\gamma M_\infty^2 \varepsilon(x)}{1 - M_\infty^2}, \qquad \rho' = \frac{\varepsilon(x)}{1 - M_\infty^2},$$

$$u' = -\frac{\varepsilon(x)}{1 - M_\infty^2}, \qquad T' = \frac{(\gamma M_\infty^2 - 1)\,\varepsilon(x)}{M_\infty^2 - 1}, \tag{8}$$

where M_∞ is the Mach number of the flow for ε = 0, and γ is the adiabatic constant. The figure shows the results of calculations obtained on a computer by V. N. Makarov (the broken lines) and using Eqs. (3)-(8) (the continuous lines) for the same flow conditions. The initial data were as follows: a mixture of 15% CO_2 + 83% N_2 + 2% H_2O, in the forechamber

---

†The collisional deactivation in the modes i = 3, 4 become important after the radiation intensity becomes zero when the condition k^* = const is in fact no longer satisfied.

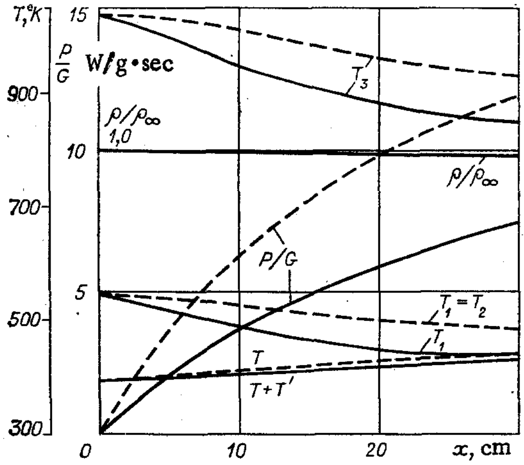

Fig. 1

of the nozzle p = 15 atm and T = 2060°K, at the resonator input $u_\infty = 2\cdot10^5$ cm/sec, $p_\infty/p = 10^{-4}$, $\rho_\infty/\rho = 5.10^{-3}$, T_∞ = 393 °K, $k_\infty = 0.004809$ cm^{-1}, in the resonator $k^* = 0.000799$ cm^{-1}, d = 80 cm, L = 30 cm, t = 0.1, and $\alpha = 0.02$ for two transits of the beam, and the ratio of the height of the nozzle output to the critical value $S_1 = 53.83$. Within the framework of models considered [1-6] the agreement must be regarded as satisfactory. For calculations of the profiles of the temperatures T_1 and T_3 we used the rate constants given in [11].

The author thanks V. N. Makarov for providing the theoretical data for comparison and for useful discussions.

LITERATURE CITED

1. I. A. Generalov, G. I. Kozlov, and I. K. Selezneva, "Calculation of the characteristics of a gas flow laser," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1972).
2. S. A. Losev and V. N. Makarov, "The power of a gas flow laser at high pressure," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1975).
3. T. A. Cook, Power and gain characteristic of high speed flow lasers," J. Appl. Phys., 40, No. 9 (1969).
4. A. L. Mikaelyan, V. P. Minaev, Yu. A. Obod, and Yu. G. Gurkov, "The characteristics of a CO_2 laser with transverse pumping operating in the gain mode," Kvantovaya Elektron., No. 5 (1974).
5. V. K. Konyukhov, "Gas flow CO_2 lasers," Fiz. Inst. Akad. Nauk Preprint, No. 141 (1976).
6. A. I. Anan'kin, "Calculation of the electrical characteristics of gas flow lasers," Vopr. Radioelektron., Ser. Obshch. Tekh., No. 1 (1976).
7. V. M. Kuznetsov, "Population inversion of the oscillatory levels of molecules near bodies in hypersonic flow," Uch. Zap. TsAGI, 5, No. 6 (1973).
8. S. A. Losev, Gas Flow Lasers [in Russian], Nauka, Moscow (1977).
9. V. M. Kuznetsov, "Oscillatory relaxation of multiatomic molecules in a field of monochromatic radiation behind a shock front," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1977).
10. V. N. Zhigulev, "The problem of the flow of a nonequilibrium gas," Dokl. Akad. Nauk SSSR, 149, No. 6 (1963).
11. R. L. Taylor and S. Bitterman, "Survey of vibrational relaxation data for processes important in the CO_2–N_2 laser system," Rev. Mod. Phys., 41, No. 1 (1969).